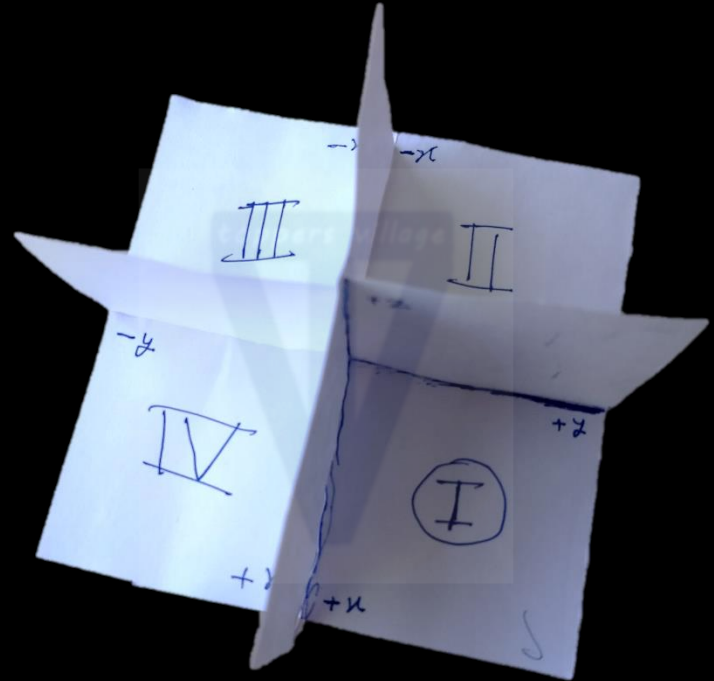
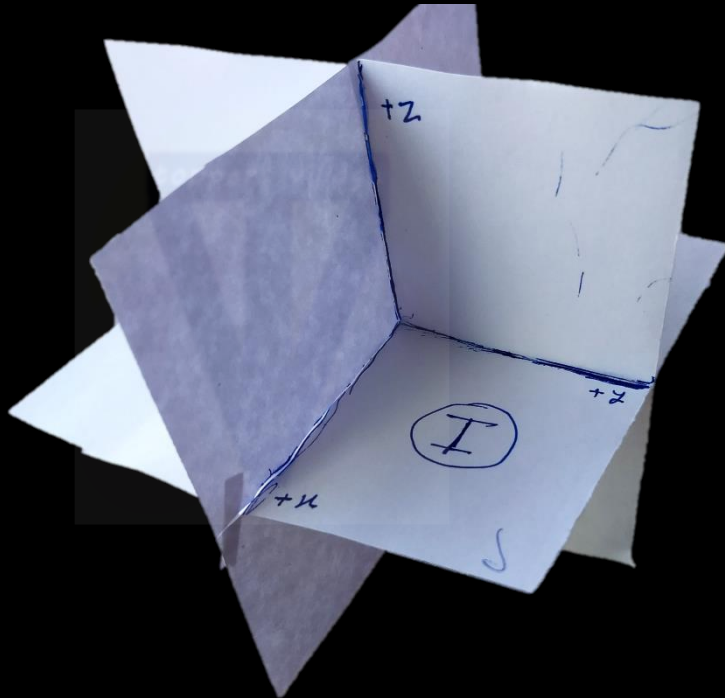




Class 11 Maths || Chapter 12 : Introduction to Three Dimensional Geometry

|| Class Notes



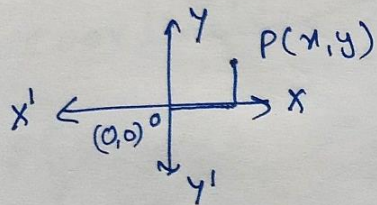
YouTube पर इस Chapter के FREE Videos देखिए (Playlist की Link नीचे है)

<https://www.youtube.com/watch?v=cl8Zi0ImxJc&list=PLPFrn0ppwwkQoKQ4ooVNipBQzyddeF8bb>

Chapter - 12 Introduction to 3D Geometry



A Brief Revision of 2D



On x-axis $\rightarrow (x, 0)$

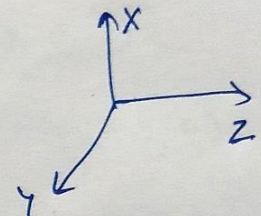
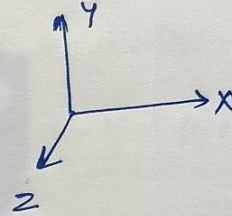
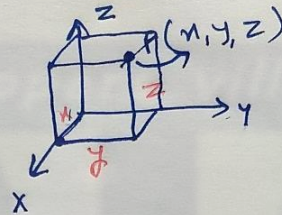
On y-axis $\rightarrow (0, y)$

x-coordinate $\rightarrow \oplus \rightarrow$ Right
 $\rightarrow \ominus \rightarrow$ Left

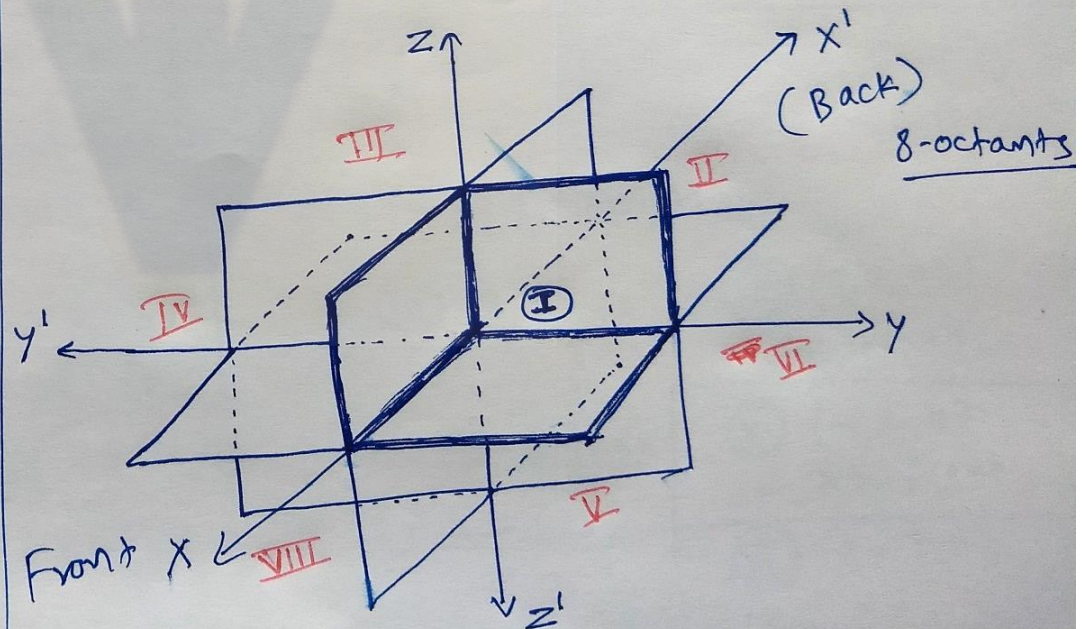
y-coordinate $\rightarrow \oplus \rightarrow$ Up
 $\rightarrow \ominus \rightarrow$ Down

4 - Quadrants

Three Dimensional Geometry (3D)



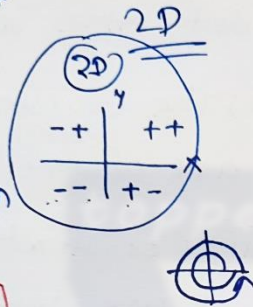
Right Hand Thumb Rule



x-coordinate $\begin{cases} \oplus & \text{Front} \\ \ominus & \text{Back} \end{cases}$

y-coordinate $\begin{cases} \oplus & \text{Right} \\ \ominus & \text{Left} \end{cases}$

z-coordinate $\begin{cases} \oplus & \text{UP} \\ \ominus & \text{Down} \end{cases}$



$P(x, y, z)$

8-Octants

	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

Special Cases.

Origin $O(0, 0, 0)$

on x-axis $\rightarrow (x, 0, 0)$

on y-axis $\rightarrow (0, y, 0)$

on z-axis $\rightarrow (0, 0, z)$

on XY-plane $\rightarrow (x, y, 0)$

on YZ-plane $\rightarrow (0, y, z)$

on ZX-plane $\rightarrow (x, 0, z)$

e.g. Identify the position of the following points.

① $(-2, 7, -13)$

⑥ $(-2, 7, -13)$
 $\downarrow$
VI-Octant

② $(-1, -2, -3)$

⑦ $(-1, -2, -3)$
 $\downarrow$
VII-Octant

③ $(2, 2, 4)$

① $(2, 2, 4)$
 $\downarrow$
II-Octant

④ $(7, 0, -5)$

⑧

XZ-plane

⑤ $(-1, 2, 0)$

②

XY-plane

⑥ $(0, -3, 0)$

④

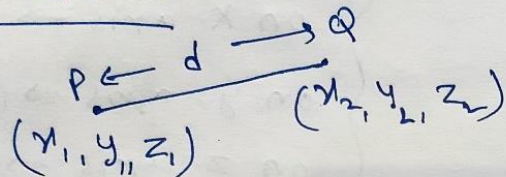
y-axis

⑦ $(1, 2, -3)$

Comment?

3D - Geometry - All Basic Formulas

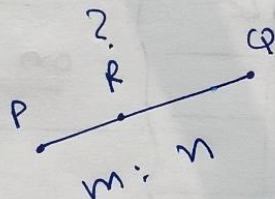
① Distance Formula



$$d = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

② Section Formula

Internal Division

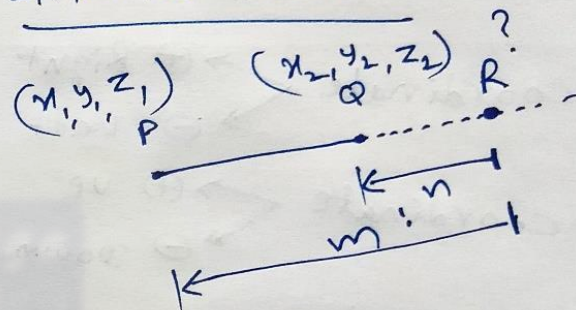


$$P(x_1, y_1, z_1)$$

$$Q(x_2, y_2, z_2)$$

$$R\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

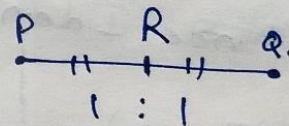
External Division



$$PR : QR = m : n$$

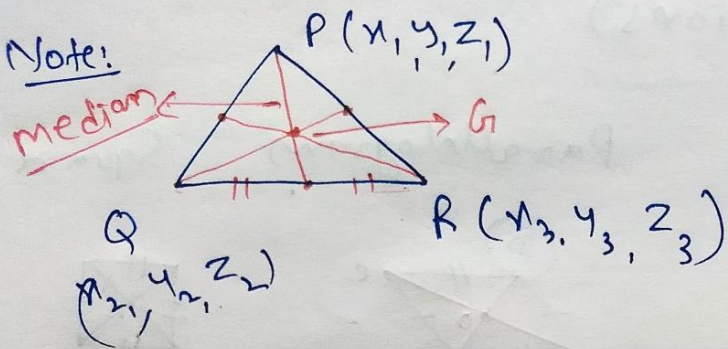
$$R\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Note: Mid Point Formula



$$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

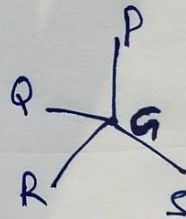
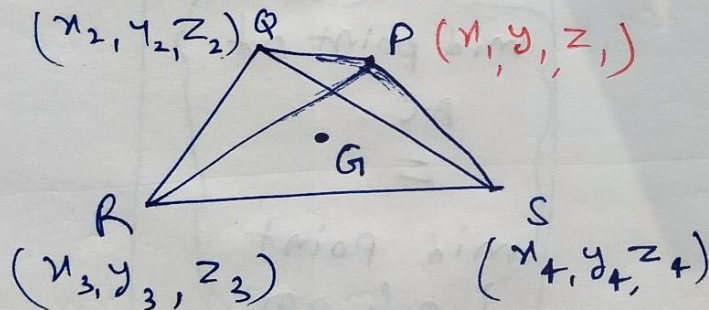
Note:



Centroid (G) of Triangle $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$

Note.

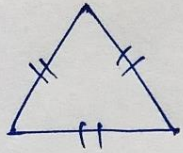
Tetrahedron.



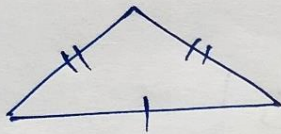
Centroid (G): $\left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4} \right)$

Basic Concepts (to Deal with Questions)

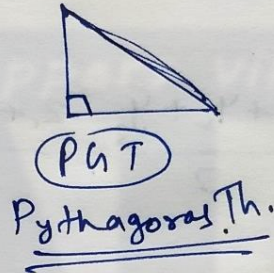
EQUILATERAL
Triangle



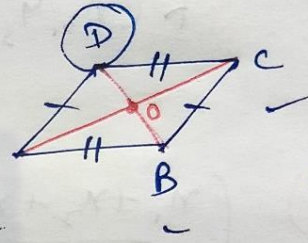
Isosceles
Triangle



Right angled
Triangle



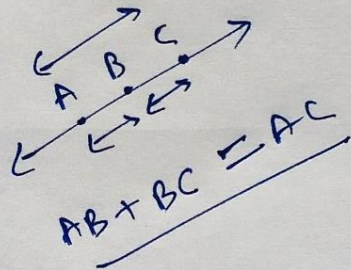
Parallelogram



Square



Collinear Points.



find 4th vertex

mid point of
AC
=
mid point of
BD

Toppers Village

Exercise 12.1 (3D) Class-11-maths

Q.1 A point on X-axis.

$$(x, 0, 0)$$

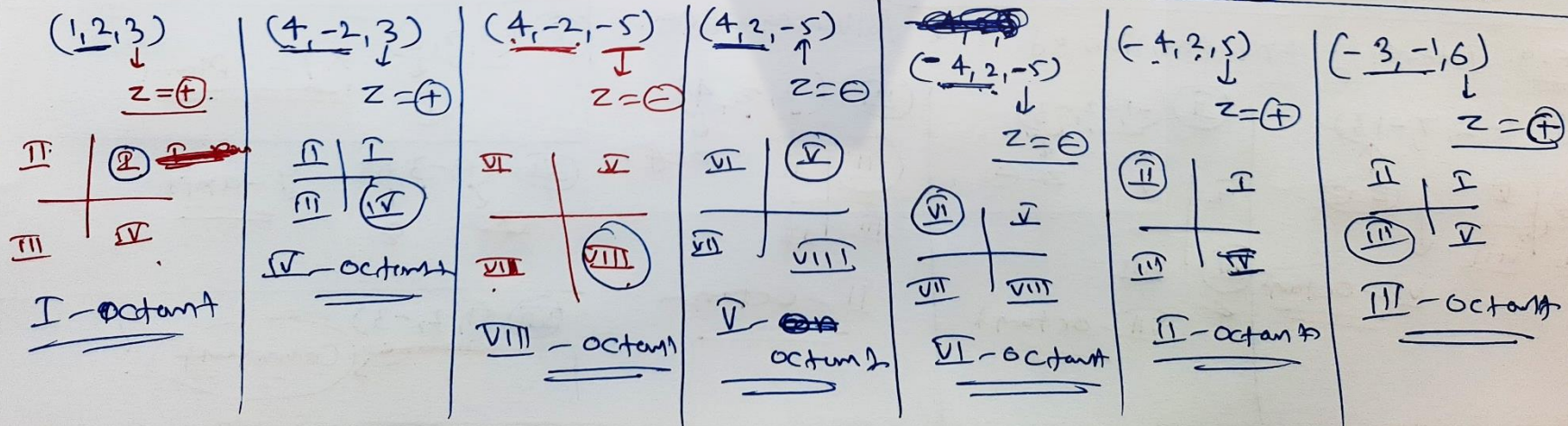
$$y\text{-coordinate} = 0$$

$$z\text{-coordinate} = 0$$

Q.2 A point in the XZ-plane

$$(x, 0, z) \quad y\text{-coordinate} = 0$$

Q.3 Name the octants.



Q.4 Fill in the Blanks:

- (i) The x-axis and y-axis taken together determine a plane known as XY-plane.
- (ii) The coordinates of points in the XY-plane are of the form $(x, y, 0)$.
- (iii) Coordinate planes divide the Space into 8 octants.

$$(2, -4, -7)$$

$$z = (-)$$

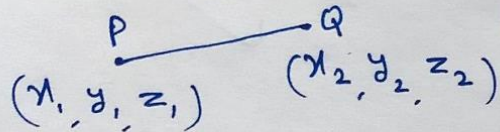
$$\begin{array}{c|c} \text{VI} & \text{V} \\ \hline \text{VII} & \text{VIII} \end{array}$$

$$\text{VIII} \rightarrow \text{VIII - Octant}$$

Toppers Village

Class - 11 - Maths Exercise (12.2)

Distance Formula.



$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

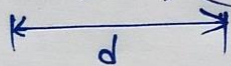
(ii) $(-3, 7, 2)$ & $(2, 4, -1)$

$$d = \sqrt{(-3-2)^2 + (7-4)^2 + (2+1)^2}$$

$$d = \sqrt{25 + 9 + 9}$$

$$d = \sqrt{43}$$

Q.1 (i) $(2, 3, 5)$ & $(4, 3, 1)$



$$d = \sqrt{(2-4)^2 + (3-3)^2 + (5-1)^2}$$

$$d = \sqrt{4 + 0 + 16}$$

$$d = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

(iii) $(-1, 3, -4)$ & $(1, -3, 4)$

$$d = \sqrt{(-2)^2 + (6)^2 + (-8)^2}$$

$$d = \sqrt{4 + 36 + 64}$$

$$d = \sqrt{104} = \sqrt{4 \times 26}$$

$$d = 2\sqrt{26} \checkmark$$

(iv) $(2, -1, 3), (-2, 1, 3)$

$$d = \sqrt{4^2 + (-2)^2 + (3-3)^2}$$

↓
0

$$d = \sqrt{16 + 4 + 0}$$

$$d = \sqrt{20} = 2\sqrt{5} \checkmark$$

$$\begin{aligned} PQ &= \sqrt{14} \\ QR &= 2\sqrt{14} \\ RP &= 3\sqrt{14} \end{aligned}$$

$$PQ + QR = RP$$

$\therefore P, Q, R \rightarrow \text{Collinear}$

[Q.2] Show that $P(-2, 3, 5), Q(1, 2, 3)$ & $R(7, 0, -1)$ are collinear.

Proof: $PQ = \sqrt{(-3)^2 + (1)^2 + 2^2} = \sqrt{14}$

$$QR = \sqrt{(-6)^2 + (2)^2 + (3+1)^2} = \sqrt{56} = \sqrt{4 \times 14} = 2\sqrt{14}$$

$$RP = \sqrt{(9)^2 + (-3)^2 + (-6)^2} = \sqrt{126} = \sqrt{9 \times 14} = 3\sqrt{14}$$

③ Verify.

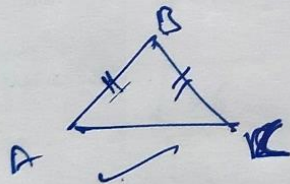
(i) $(0, 7, -10)$ A
 $(1, 6, -6)$ B
 $(4, 9, -6)$ C } Isosceles Triangle

$$AB = \sqrt{\underbrace{(-1)^2}_{+} + \underbrace{(1)^2}_{+} + \underbrace{(-4)^2}_{+6}} = \sqrt{18}$$

$$BC = \sqrt{\underbrace{(-3)^2}_{\downarrow 9} + \underbrace{(-3)^2}_{\downarrow 9} + \underbrace{(0)^2}_{0}} = \sqrt{18}$$

$$CA = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\underline{AB = BC}$$



Isosceles Δ

(ii) $(0, 7, 10)$ A
 $(-1, 6, 6)$ B
 $(-4, 9, 6)$ C } Right angled Triangle

PGT

$$AB = \sqrt{(1)^2 + (1)^2 + (4)^2} = \sqrt{18}$$

$1 + 1 + 16$

$$BC = \sqrt{(3)^2 + (-3)^2 + 0} = \sqrt{18}$$

$$AC = \sqrt{(4)^2 + (-2)^2 + (4)^2} = \sqrt{36}$$

$16 + 4 + 16$

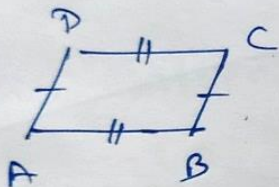


$$\begin{array}{l|l} AB^2 = 18 & AC^2 = 36 \\ BC^2 = 18 & \end{array}$$

$$\underline{AB^2 + BC^2 = 36 = AC^2}$$

PGT $\therefore$ Right angled triangle

- ③ (iii) $\begin{Bmatrix} (-1, 2, 1) & A \\ (1, -2, 5) & B \\ (4, -7, 8) & C \\ (2, -3, 4) & D \end{Bmatrix} \rightarrow \text{Vertices of Parallelogram}$



$\underline{AB = CD} \text{ \& } \underline{BC = AD}$

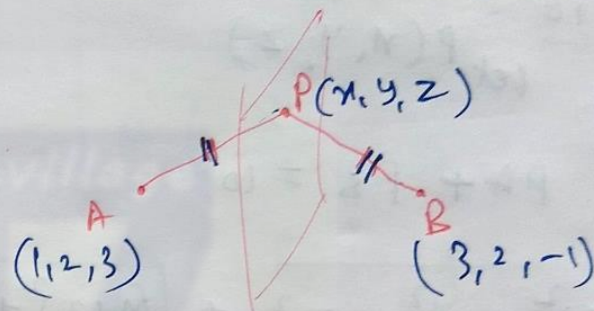
$$AB = \sqrt{4 + 16 + 16} = \underline{\underline{\sqrt{36}}}$$

$$BC = \sqrt{9 + 25 + 9} = \underline{\underline{\sqrt{43}}}$$

$$CD = \sqrt{4 + 16 + 16} = \underline{\underline{\sqrt{36}}}$$

$$AD = \sqrt{9 + 25 + 9} = \underline{\underline{\sqrt{43}}}$$

set of
[Q.4] A Points equidistant from the
Points $A(1, 2, 3)$ & $B(3, 2, -1)$.



Equidistant .

$$PA = PB$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

$$= \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9$$

$$= x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

$$\Rightarrow 6x - 2x = 2z + 6z$$

$$\Rightarrow 4x = 8z \Rightarrow \boxed{x = 2z} \leftarrow$$

Q.5

$$A(4, 0, 0)$$

$$B(-4, 0, 0)$$

$$\text{Sum} = 10$$

$$\text{let } P(x, y, z)$$

ATQ.

$$PA + PB = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \left(\sqrt{x^2 - 8x + 16 + y^2 + z^2} \right)^2 = \left(10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2} \right)^2 \quad (a-b)^2$$

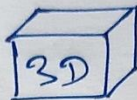
$$\Rightarrow \cancel{x^2} - 8\cancel{x} + 16 + \cancel{y^2} + \cancel{z^2} = 100 + \cancel{x^2} + 8\cancel{x} + 16 + \cancel{y^2} + \cancel{z^2} - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow \frac{-100 - 16x}{-4} = \frac{-20\sqrt{x^2 + y^2 + z^2 + 8x + 16}}{-4}$$

$$\Rightarrow [25 + 4x]^2 = \left(5 \cdot \sqrt{x^2 + y^2 + z^2 + 8x + 16} \right)^2$$

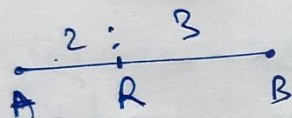
$$\Rightarrow 625 + 16x^2 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$



Q.1 $A(-2, 3, 5)$ $B(1, -4, 6)$

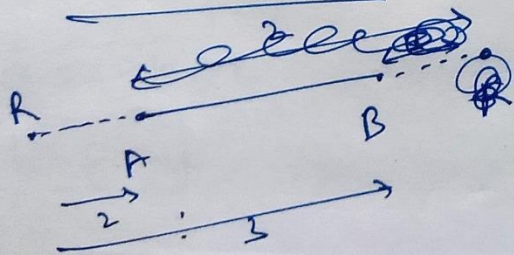
(i) 2:3 internally



$$R \left(\frac{2 \times 1 + 3 \times (-2)}{2+3}, \frac{-8 + 9}{5}, \frac{12 + 15}{5} \right)$$

$$R \left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5} \right)$$

(ii) 2:3 externally



externally

$$R \left(\frac{2 - (-6)}{2-3}, \frac{-8-9}{2-3}, \frac{12-15}{2-3} \right)$$

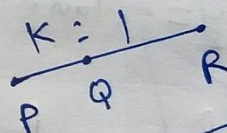
$$R \left(\frac{8}{-1}, \frac{-17}{-1}, \frac{-3}{-1} \right)$$

$$R(-8, 17, 3)$$

Q.2 $P(3, 2, -4)$ $Q(5, 4, -6)$

$$R(9, 8, -10)$$

collinear



$$\frac{PQ}{QR} = ? = K:1$$

Let

By Section Formula,

$$\begin{aligned} 5 &= \frac{K \cdot 9 + 1 \times 3}{(K+1)} \Rightarrow 5K+5 = 9K+3 \\ \text{(Given)} \end{aligned}$$

$$\Rightarrow 5K + 5 = 9K + 3$$

$$\Rightarrow 5 - 3 = 9K - 5K$$

$$\Rightarrow 2 = 4K$$

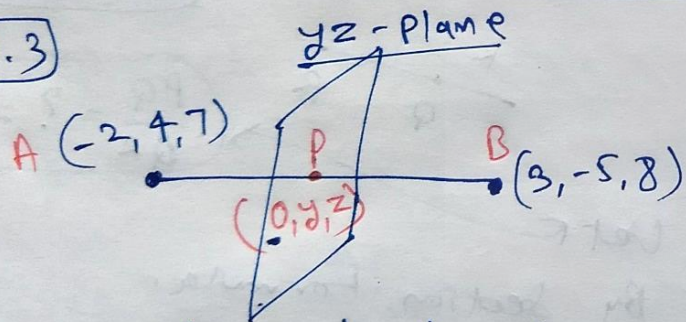
$$\Rightarrow 1 = 2K$$

$$\Rightarrow K = \frac{1}{2}$$

$$K:1 = \left(\frac{1}{2}\right):1 = \frac{\left(\frac{1}{2}\right)}{(1)} = \frac{1}{2}$$

$$\text{Ratio} = 1:2 \quad \checkmark$$

[Q.3]



$$K:1 \text{ (let)}$$

By Section Formula,

$$\text{x-coordinate of P} = \frac{3K + 1(-2)}{K+1}$$

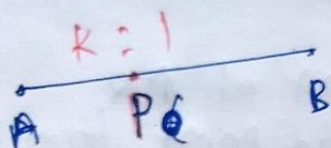
$$\Rightarrow 0 = \frac{3K - 2}{K+1}$$

$$\Rightarrow 0 = 3K - 2$$

$$\Rightarrow 2 = 3K \Rightarrow \boxed{K = \frac{2}{3}}$$

$$\text{Required Ratio} = \frac{K}{1} = \frac{2}{3}$$

Q.4 $A(2, -3, 4)$ are collinear
 $B(-1, 2, 1)$
 $C(0, \frac{1}{3}, 2)$ (Using Section Formula)



Let P divides AB in ratio $K:1$

$$P\left(\frac{-K+2}{K+1}, \frac{2K-3}{K+1}, \frac{K+4}{K+1}\right)$$

Let P & C coincide,

x-coordinate of C = x-coord. of P

$$\Rightarrow 0 = \frac{-K+2}{K+1}$$

$$\Rightarrow 0 = -K+2$$

$$\Rightarrow \boxed{K=2}$$

Put $K=2$ in coordinates of P.

$$P\left(0, \frac{1}{3}, \frac{6}{3}\right)$$

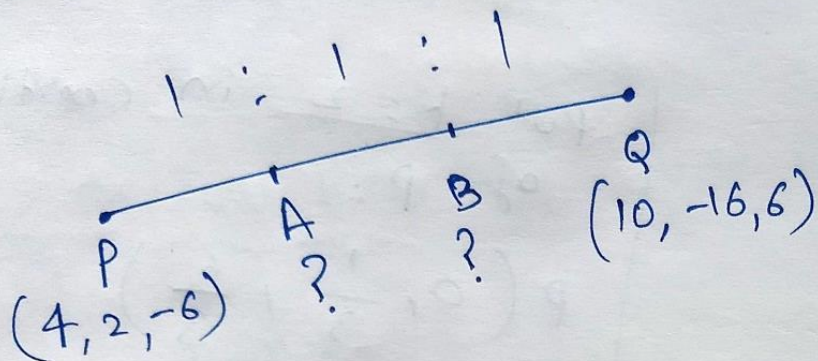
$$P\left(0, \frac{1}{3}, 2\right) \leftrightarrow C$$

$\therefore C$ lies on the line segment AB.

$\therefore A, B, C \rightarrow$ collinear.

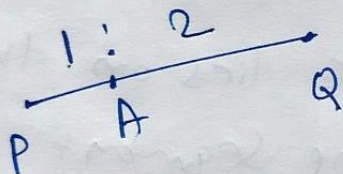
(C divides AB in 2:1 internally)

⑤



$$PA = AB = BQ$$

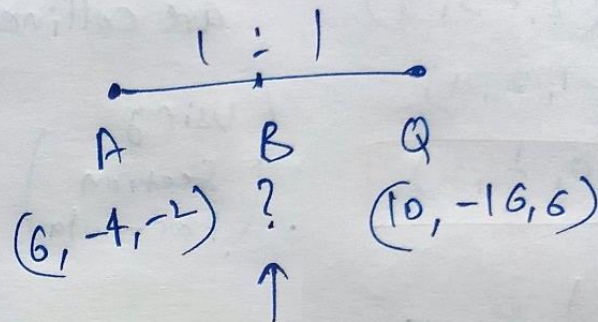
A



By Section Formula

$$A\left(\frac{10+8}{1+2}, \frac{-16+4}{1+2}, \frac{6-12}{1+2}\right)$$

$$\Rightarrow A(6, -4, -2) \checkmark$$



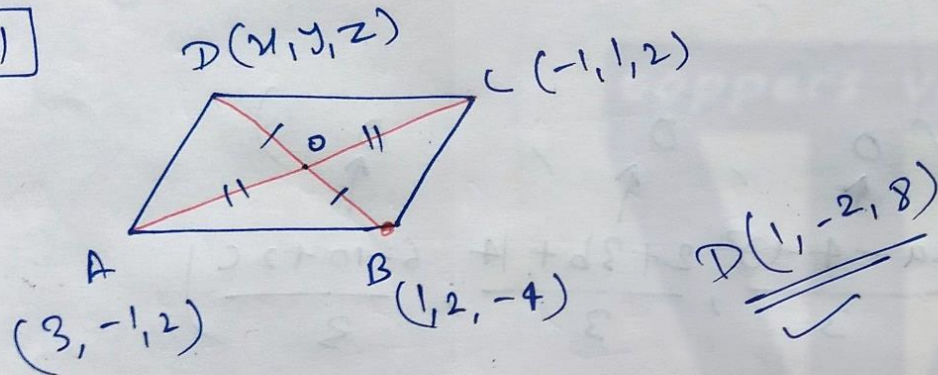
mid point of AQ
(mid point formula)

$$B\left(\frac{16}{2}, \frac{-20}{2}, \frac{4}{2}\right)$$

$$B(8, -10, 2)$$

Miscellaneous Exercise - Chapter 12

Q.1



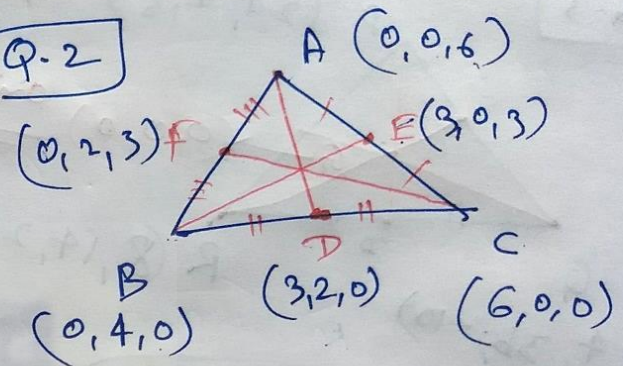
Diagonals bisect each other.

$\therefore$ mid point of AC
= mid point of BD.

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) \equiv \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2} \right)$$

$$\begin{aligned} 2 &= 1+x & 0 &= 2+y & 4 &= -4+z \\ \boxed{x=1} & & \boxed{y=-2} & & \Rightarrow \boxed{z=8} \end{aligned}$$

Q.2



mid point formula

$$D \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) \equiv D(3, 2, 0)$$

$$E(3, 0, 3)$$

$$F(0, 2, 3)$$

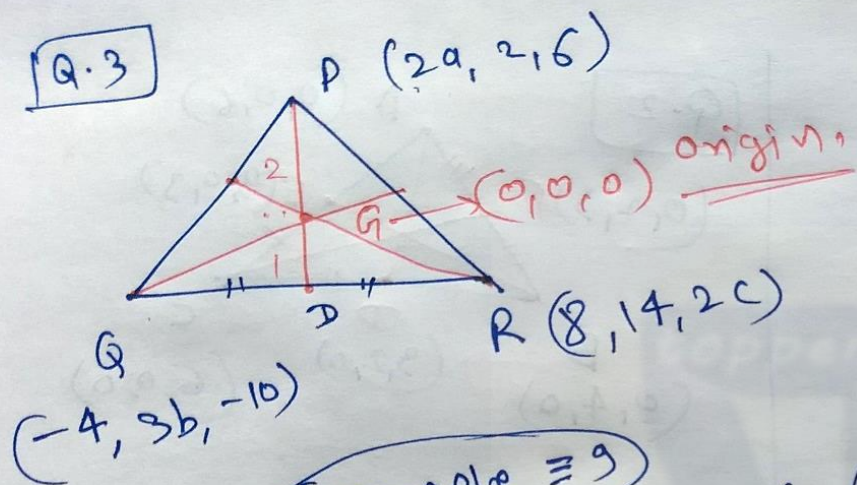
Median (Distance Formula)

$$AD = \sqrt{9+4+36} = \sqrt{49} = 7 \checkmark$$

$$BE = \sqrt{9+16+9} = \sqrt{34} \checkmark$$

$$CF = \sqrt{36+4+9} = 7 \checkmark$$

Q.3



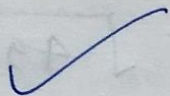
Example $\equiv 9$

$$G \left(\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right)$$

$$\text{Centroid of } \triangle PQR \equiv G \left(\frac{2a - 4 + 8}{3}, \frac{2 + 3b + 14}{3}, \frac{6 - 10 + 2c}{3} \right)$$

$$\Rightarrow \frac{2a + 4}{3} = 0$$

$$\Rightarrow \boxed{a = -2}$$



$$\left\{ \begin{matrix} \frac{3b + 16}{3} = 0 \\ \boxed{b = -\frac{16}{3}} \end{matrix} \right.$$



$$\left\{ \begin{matrix} \frac{2c - 4}{3} = 0 \\ \boxed{c = 2} \end{matrix} \right.$$



Q.4

$P(3, -2, 5)$

A point on y-axis $A(0, y, 0)$

$$d = 5\sqrt{2}$$

ATQ. $AP = d$

$$\Rightarrow AP^2 = d^2$$

$$\Rightarrow (3)^2 + (-2-y)^2 + (5)^2 = (5\sqrt{2})^2$$

$$\Rightarrow 9 + 4 + y^2 + 4y + 25 = 50$$

$$\Rightarrow y^2 + 4y - 12 = 0$$

$$\Rightarrow y^2 + 6y - 2y - 12 = 0$$

$$\Rightarrow y(y+6) - 2(y+6) = 0$$

$$\Rightarrow (y-2)(y+6) = 0$$

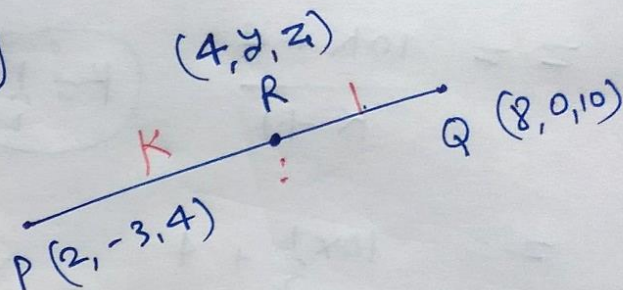
$y = 2, y = -6$

Answer

$A(0, 2, 0)$ ✓

$A(0, -6, 0)$ ✓

Q.5



Section Formula,

$$R\left(\frac{8K+2}{K+1}, \frac{-3}{K+1}, \frac{10K+4}{K+1}\right)$$

But $R(4, y, z)$

Comparison:

$$\frac{8K+2}{K+1} = 4 \Rightarrow 8K+2 = 4K+4$$

$$\Rightarrow 4K = 2 \Rightarrow K = \frac{1}{2}$$

$$y = \frac{-3}{K+1} = \frac{-3}{\frac{1}{2}+1} = \frac{-3}{\frac{3}{2}} = -2$$

$$y = -2$$

$$z = \frac{10K+4}{K+1}$$

$$K = \frac{1}{2}$$

$$z = \frac{10 \times \frac{1}{2} + 4}{\frac{1}{2} + 1} = \frac{5+4}{\left(\frac{3}{2}\right)} = \frac{9}{\left(\frac{3}{2}\right)} = 6$$

$$z = 6$$

$$R(4, -2, 6)$$

$$\Rightarrow x^2 + 9 - 6x + y^2 - 8y + 16 + z^2 + 25 - 10z + x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 49 + 14z = K^2$$

$$\Rightarrow \boxed{2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = K^2}$$

$$\boxed{\text{Q.6}} \quad A(3, 4, 5) \text{ \& } B(-1, 3, -7)$$

$$P(x, y, z)$$

$$PA = \sqrt{\quad} \quad PB = \sqrt{\quad}$$

Given: $(PA)^2 + (PB)^2 = K^2$

$$\Rightarrow (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$+ (x+1)^2 + (y-3)^2 + (z+7)^2 = K^2$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>